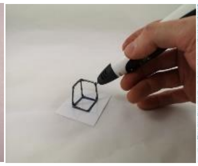
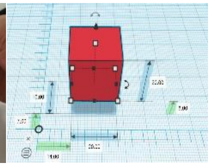
International Symposium on 3D Printing in Mathematics Education

September 16-17 2021



The use of 3D printing technology to support the learning of mathematics has been of increasing interest among teachers and in the research community in the last few years. Many fascinating ideas have been realized and important scientific findings have been made. However, there is still a great need for more concrete ideas, realizations and further scientific research. To fill this desideratum, the collaboration of practice and research is essential. With this international symposium we want to contribute to the scientific discourse by providing a forum for the presentation of existing insights and by establishing a network of involved parties from research and practice.

Please send questions and comments to the following email address: 3DPrinting@mathematik.uni-siegen.de

The symposium will take place on Zoom. Please not the Dial-in data below.

Organizing Team:
Frederik Dilling,
Dr. Felicitas Pielsticker
& Prof. Dr. Ingo Witzke

Universität Siegen

Guide for the International Symposium on 3D Printing in Mathematics Education

Schedule

Time according to "Europe/Berlin" time zone.

Thursday September 16

9:00	Opening (Information on the conference)
9:30-10:15	Opening presentation (Prof. Dr. Ingo Witzke) *3D-Printing in Mathematics Education - An Overview of Five Years of Research in and for the Classroom*
10:15-10:30	Break
10:30-12:00	1. Presentation block (Four presentations in two parallel sessions)
12:00-12:15	Break
12:15-13:45	2. Presentation block (Four presentations in two parallel sessions)
13:45-14:30	Lunch break
14:30-16:00	3. Presentation block (Four presentations in two parallel sessions)
16:00-16:15	Conclusion and outlook to the next day

Friday September 17

9:00	Open beginning/ Meet & Greet
9:30-11:00	4. Presentation block (Four presentations in two parallel sessions)
11:00-11:15	Break
11:15-12:00	5. Presentation block (Two parallel presentations)
12:00-12:30	Conclusion (outlook, possibilities for publication)

The symposium will take place on the platform Zoom. You can find the dial-in data in the following:

International Symposium on 3D Printing in Mathematics Education
Time: 16th Sept..2021 09:00 AM Amsterdam, Berlin, Rome, Stockholm, Vienna
17th Sept..2021 09:00 AM
Zoom-Meeting
https://uni-siegen.zoom.us/j/92706186617?pwd=WDZpZFFQeVpjLzZXNnlMUENWSUdqQT09

Meeting-ID: 927 0618 6617
Identification Code: AD#?Ca7B
Quick Dial-Up by Mobile Phone
+496950502596,,92706186617#,,,,*45656892# Deutschland
+496971049922,,92706186617#,,,,*45656892# Deutschland

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+43 670 309 0165 Österreich
+1 646 876 9923 Vereinigte Staaten (New York)
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162.255.37.11 (USA West)
162.255.36.11 (USA East)
213.19.144.110 (Amsterdam Netherlands)
213.244.140.110 (Germany)
Identification Code: 45656892
Meeting-ID: 927 0618 6617

Overview Presentations

Thursday September 16

1. Presentation block

10:30-11:15

Frederik Dilling

3D-Printing in Calculus Education – Concrete Ideas and Research Results

Dr. Gero Stoffels & Dr. Felicitas Pielsticker

Interfaces in Learning Mathematics Using 3D Printing Technology – Challenging and Encouraging Visualizations Switching 3D↔2D

11:15-12:00

Dr. Rudolf Hrach

Plane Tessellations

Dr. Tim Lutz

3D Printing Solutions - An Overview of materials developed for Primary School to High School up to Higher Education (teacher education)

2. Presentation block

12:15-13:00

Frederik Dilling, Dr. Gregor Milicic & Amelie Vogler

Coding and 3D-Printing

Eva Ulbrich & Branko Andjic

Possibilities and Obstacles of 3D printing in Lessons

13:00-13:45

Robert Päßler, Margarete Ketelsen & Prof. Dr.-Ing. Daniel Lordick

Maistaeder

Dr. Felicitas Pielsticker & Birgitta Marx

"That's not fair" – Negotiating the meaning of the concept of fairness with 3D printing in an 10th grade class

3. Presentation block

14:30-15:15

Gustavo Aguilar & Mathías Tejera

Modeling and 3D printing in the maths class and a way to develop STEAM projects

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Denish Akuom, Jessica Davidson, Erin Pomponio, Prof. Dr. Eileen Fernandez, ao.Prof. Dr. Steven Greenstein

The Personal, Pedagogical, And Problem-Solving Possibilities Of Mathematical Making In Teacher Preparation

15:15-16:00

Dr. Diego Lieban & Lab Team

Math and Chemistry together: modeling 3D molecules

Mira H. Wulff, Prof. Dr. Aiso Heinze & Dr. Marc Wilken

DiASper - Increasing the skills on occupationally relevant digital technologies among young people in Southern Denmark and the state of Schleswig-Holstein (Northern Germany)

Friday September 17

4. Presentation block

9:30-10:15

Jenny Knöppel & Dr. Felicitas Pielsticker

Using the 3D printing technology for mathematical reasoning – an empirical learning environment on the "Lunes of Hippocrates"

Shereen ElBedewy

3D transformations for Architectural models as a tool for mathematical learning

10:15-11:00

Dr. Rudolf Hrach

Platonic solids as edge-models

Dr. Oi-Lam Ng

Doing Mathematics with 3D Pens: Five Years of Research on 3D Printing Integration in Mathematics Classrooms

5. Presentation block

11:15-12:00

Frederik Dilling & Amelie Vogler

Designing Mathematical Drawing Instruments with 3D-Printing

Eva Ulbrich, Julia Handl, Shereen ElBedewy, Martin Mayerhofer & Branko Andjic

Three dimensional mathematics to aMaze

Abstracts

Prof. Dr. Gustavo Aguilar[1] & Prof. Dr. Mathías Tejera[2]
[1] IPA - JKU - Goegebra Uruguay (Uruguay)
[2] UdelaR (Uruguay)

Modeling and 3D printing in the maths class and a way to develop STEAM projects

Within the mathematics teacher's community, there is a consensus about the need to include technology in the classroom, but there is not, about how to do it. In our experience, most mathematics teachers struggle with the belief that they need to master the tool before including it in a classroom activity. With the intention to help knocking out that -from our perspective- misconception is that we present this work to introduce the idea, tips and ways of working with 3D modeling and 3D printing in the maths class as part of STEAM projects. By modeling real life structures using functions and equations we can create printable objects in GeoGebra that can be printed in any 3D printer. This 3D models allow us to use many calculus tools and applied mathematics. In this direction, Hoyles and Lagrange (2010) underlines the impact of the tool used, in the link between the user and de phenomenon, shaping the ways students do and think about mathematics. This helps us create links between maths and real life thus, providing richer meanings of math objects and creating a good opportunity to develop STEAM projects. In this presentation we pretend to show how to create models using the GeoGebra

software and give advise on how to use GeoGebra to create 3D printable surfaces using some basic GeoGebra commands and mathematics. Also this STEAM projects can be adapted to many levels of mathematics and we can create the opportunity to include historical or cultural aspects into the mathematics class.

Denish Akuom, Jessica Davidson, Erin Pomponio, Prof. Dr. Eileen Fernandez & ao.Prof. Dr. Steven Greenstein, Montclair State University (USA)

The Personal, Pedagogical, And Problem-Solving Possibilities Of Mathematical Making In Teacher Preparation

In this proposal, we aim to explore how the designing of 3-D printed tools by prospective elementary mathematics teachers (PMTs) acts as a creative process that reveals the possibilities of mathematical Making for teacher learning. We offer analyses of concrete manipulatives designed by PMTs in order to elucidate how students' lived histories, content and pedagogical intentions, or dynamic knowings are leveraged in their design, fabrication, and use. Our analyses reveal three distinct possibilities that may emerge during the Making process: personal, pedagogical, and problem solving. These findings illustrate opportunities for multi-dimensional engagement centered around a 3-D printed mathematical manipulative by teacher educators, PMTs, and their students. Implications for the promise of a Making experience in supporting meaningful mathematical learning within teacher preparation are discussed.

Frederik Dilling, University of Siegen (Germany)

3D-Printing in Calculus Education – Concrete Ideas and Research Results

The importance of qualitative approaches to the central topics of calculus has been discussed for a long time. 3D printing technology offers many opportunities to address this demand by working with CAD software or printed manipulatives. In this talk, some concrete ideas as well as research results from several studies will be presented.

Frederik Dilling[1], Dr. Gregor Milicic[2] & Amelie Vogler[1]
[1]University of Siegen (Germany)
[2]Federal Ministry of Education and Research (Germany)

Coding and 3D-Printing

This presentation will address programming in the context of 3D printing. The focus is on the block programming function of the computer-aided design software BlocksCAD, which allows the formulation of algorithms for the development of three-dimensional objects by arranging code blocks as graphical elements. This opens up various applications in the field of mathematics. Three examples will be presentet in the talk.

Frederik Dilling & Amelie Vogler, University of Siegen (Germany)

Designing Mathematical Drawing Instruments with 3D-Printing

Mathematical drawing instruments play a special role in the history of mathematics and can also be used meaningfully in learning processes in the mathematics classroom. 3D printing technology offers the possibility to develop drawing instruments for the students or to let the students develop the devices in the classroom themselves. This will be explained in the presentation using the example of an integraph and a pantograph. A suitable theoretical background will be presented and two empirical studies will be referred to.

Shereen ElBedewy, Johannes Kepler Universität Linz (Egypt)

3D transformations for Architectural models as a tool for mathematical learning

The idea behind the presentation is to use architectural models and the various visualization technologies to transfer mathematical learning and reflect on cultural aspects.

Dr. Rudolf Hrach, Siegen (Germany)

Plane Tessellations

Polygons play an important role in school geometry. With certain polygons one can tessellate the plane (keywords: regular, semi-regular tessellation, Laves lattices). With 3D Print one can easily produce a lot of them. Artists as Vasarely and Escher used tessellation and this can be motivation for pupils.

Dr. Rudolf Hrach, Siegen (Germany)

Platonic solids as edge-models

The five Platonic solids are attractive subjects in space geometry. With edge–models one can study the inner region of them. If you build them as edge-models you need vertex-connectors to connect the edges (you can take barbecue skewers) at their ends. The 3D print of the vertex-connectors is described in this article. A nesting of the Platonic solids (similar to Keplers model) and further applications are added.

Jenny Knöppel & Dr. Felicitas Pielsticker, Siegen (Germany)

Using the 3D printing technology for mathematical reasoning – an empirical learning environment on the "Lunes of Hippocrates"

There is a broad consensus in the community of mathematics education that justifying mathematical statements should have a place in the classroom (Jahnke & Ufer, 2015). The following presentation provides an example of how students can explore and discuss different aspects of mathematical reasoning such as the verification of a statement, explanation and discovery (de Villiers, 1990; Meyer & Prediger, 2009) by using the 3D printing technology.

In a unit on the "Lunes of Hippocrates" in grade 9, students explore and test the hypothesis that the area of a rectangular triangle equals the area of the two respective crescent moons in an empirical learning environment, by creating a 3D-model of the lunes in a CAD-program and conducting experiments with the 3D-printed objects (measuring the filament, weighing the 3D-printed objects). The empirical setting offers students the opportunity to experience and discuss the advantages as well as the limitations of experiments and to discover the importance of further functions of reasoning that go beyond verification – such as the explanation of knowledge in a more formal approach (de Villiers, 1990; Struve, 1990).

Dr. Diego Lieban & Lab Team, IFRS (Brazil)

Math and Chemistry together: modeling 3D molecules

In this talk, we will share a series of molecules, which were developed on GeoGebra by high school students and printed in 3D as part of an interdisciplinary task through a Maker Culture project. Based on a demand from chemistry, students and the lab's team are challenged to figure it out the proper atoms' ratio, angles in 3D and setting the right holes in the 3D models to develop a set of "puzzles of molecules".

Dr. Tim Lutz, University of Landau (Germany)

3D Printing Solutions - An Overview of materials developed for Primary School to High School up to Higher Education(teacher education)

An overview of various applications of current research and development assigned to the teaching and learning lab at the University of Landau. Also shown are 3D printed materials connected with AR and Machine Learning applications, which provide immediate automated feedback in real time.

Dr. Oi-Lam Ng, The Chinese University of Hong Kong (Hong Kong)

Doing Mathematics with 3D Pens: Five Years of Research on 3D Printing Integration in Mathematics Classrooms

Scholars generally agree that evolutions in technology lead to deep changes in thinking, learning and doing mathematics. In this talk, I discuss the potential changes in thinking, learning and doing that may arise from the use of "3D (Printing) Pens", which enable mathematics to be done in space, thus shifting a two-millennium old tradition of drawing on 2D sand, paper and screens. I describe the rationale for undertaking this research, some theoretical underpinnings and preliminary findings about the role of 3D Pens in the learning of geometry, functions and calculus in primary and secondary school mathematics classroom.

Robert Päßler, Margarete Ketelsen & Prof. Dr.-Ing. Daniel Lordick, TU Dresden (Germany)

Maistaeder

There are infinitely many polyhedra. However, one can classify them by their combinatorial type, a graph-theoretic invariant. For example, there are 2.606 different combinatorial types possible for polyhedra with exactly nine vertices. Polytopia.eu is a project, that allows everyone to adopt a combinatorial type and to feature its properties by crafting or 3D-printing models. Each combinatorial type has infinitely many geometric representations. Polytopia.eu has chosen the so-called Koebe-Adreev-Thurston realization for the provided polyhedral net and the 3D-model. These realizations have a sphere inscribed inside the polyhedra that touches each edge at exactly one point. In consequence, each facet has a circle inscribed that touches the edges once each. [Figure] Fig. 1: Series of Maistaeder models in the collection of mathematical models at the TU Dresden [2] We adopted the polyhedron N° 901638 [1] with the f-vector (9,18,11). We have chosen this polyhedron, because it has a nice symmetry. The name Maistaeder refers phonetically to Gustl Bayrhammer's character "Meister Eder" in "Master Eder and his Pumuckl", a German children's series created by Ellis Kaut. To appeal to certain slang sounds, the name was modified a little. First we made a cardboard model following the instructions from Polytopia.eu. Using 3D-printing with plaster, we also realized a series of edge-models [3]. Furthermore, we created models that emphasize the inscribed sphere *and the circles in the facets. This is certainly most evident when you look at the two colored models in the foreground of the image. The one on the right represents a dice with eleven facets. Using the models, several mathematical topics can be discussed instantly, e.g. combinatorics, polyhedral geometry or the circle packing problem. Resources: [1] https://www.polytopia.eu/detailansicht?id=901638 [2] https://mathematical-models.org [3] https://www.mathematical-models.org/index.php/models/view/1108*

Dr. Felicitas Pielsticker & Birgitta Marx, University of Siegen (Germany)

"That's not fair" – Negotiating the meaning of the concept of fairness with 3D printing in an 10th grade class

The focus of the study presented here is the learning environment "Chance and Fairness – Developing Ideas about the Concept of Randomness". "Randomness" is to be discovered and explored in relation to the fairness of gambling. An action-oriented approach via the digital medium of 3D printing technology with a game situation involving a dice tower supports the negotiation of the meaning of the concepts of randomness and fairness. In antiquity, a dice tower was a guarantee for the fairness of a dice game and prevented possible cheating. This raises the question of how such a dice tower must be designed to ensure that dice roll outcomes were random. It is interesting to see how students independently (further) develop and deepen the concepts of randomness and fairness and negotiate conflicts of interpretation with fellow students. The examination of the fairness of gambling and the 3D printed dice towers leads students to engage with the otherness of a historical piece of mathematics, to relate the historical view to their own and to deepen their current understanding. The learning environment was conducted in a 10th grade class as a repeat unit and each lesson was videotaped. The following question was investigated: To what extent can (students') theories of fairness be (further) developed when dealing with 3D printing technology?

This research question was described and analyzed using the descriptive approach of empirical theories.

Dr. Gero Stoffels & Dr. Felicitas Pielsticker, University of Siegen (Germany)

Interfaces in Learning Mathematics Using 3D Printing Technology – Challenging and Encouraging Visualizations Switching 3D↔2D

In this presentation of "Interfaces in Learning Mathematics Using 3D Printing Technology – Challenging and Encouraging Visualizations Switching 3D↔2D" we want to discuss a crucial interface in mathematics lessons sharing results of a concrete case study. In particular this presentation aims to show in which ways the use of 3D printing technology in the context of geometry is suitable for university teaching and answering the question to what extent this leads to meaningful opportunities for reflections of university students as well as similar reflections on geometry of school students.

Eva Ulbrich & Branko Andjic, Johannes Kepler Universität Linz (Austria)

Possibilities and Obstacles of 3D printing in Lessons

In STEAM education, 3D printing can be valuable for creating custom teaching material to create tangible visualizations of virtual concepts as are used in mathematics. We developed a workshop introducing 3D printing in ten schools to around 200 teachers for secondary. After giving teachers time to practice and utilize their new gained skills they were invited to interviews. The findings of these 37 interviews reflecting on teachers' experiences will be presented including their opinions on needs during lessons, practicability, usability, maintainability and the effect on students they perceived.

Eva Ulbrich, Julia Handl, Shereen ElBedewy, Martin Mayerhofer & Branko Andjic, Johannes Kepler Universität Linz (Austria)

Three dimensional mathematics to aMaze

We use the fun of the experience and challenge of solving mazes as motivation in a workshop that teaches 3D modeling, augmented reality and 3D printing skills. The workshop was tested both on-line and offline at the ArsElectronica outside in Kepler's garden. Participants aged five to 65 years experience the geometrical attributes of mazes to develop an understanding of mathematical concepts such as mirroring, rotation, translation and certain basics of algorithms. The created mazes from this workshop are solvable, 3D printable, shareable and experienceable using AR on mobile devices.

Prof. Dr. Ingo Witzke, University of Siegen (Germany)

3D-Printing in Mathematics Education - An Overview of Five Years of Research in and for the Classroom

For the past five years, the use of 3D printing technology in the mathematics classroom has been a focus of mathematics education research at the University of Siegen. In my talk, I will give an overview of the theoretical approaches and empirical findings of the last years which have been developed in my working group and touch perspectives for future research.

Mira H. Wulff, Prof. Dr. Aiso Heinze & Dr. Marc Wilken, IPN Kiel (Germany)

DiASper - Increasing the skills on occupationally relevant digital technologies among young people in Southern Denmark and the state of Schleswig-Holstein (Northern Germany)

One of today's central challenges is the digital transformation of the workplace, as the rapid development of digital technologies is leading to changes in professions. Consequently, innovative educational concepts are needed to prepare students for the necessary competence requirements and to ensure that they can enter the workplace with a digital-technological basic education. The DiASper project (Digital Working World from School Perspective) therefore aims at implementing aspects of digital technologies of the digital workplace in regular STEM school lessons. Taking the digital and technological processes of 3D-printing as a starting point, the project has identified learning contents that address both the curricular contents of mathematics and the digital technologies of the workplace. The technological processes of 3D-printing, namely modelling, triangulation, slicing and printing, can be used to teach mathematical content in both geometry and analysis. For instance, the use of a CAD-Software enables students of different grades to analyze compound objects by spinning the object virtually, thus providing the possibility to enhance students' spatial awareness. The project follows the design based research approach to adapt the teaching concepts for the *possibility of an optimized use in mathematics teaching.*

Authors and Participants

1. Gustavo Aguilar, IPA - JKU - Goegebra Uruguay (Uruguay)
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5. Armen Chtchian, Bielefeld University (Germany)
6. Jessica Davidson, Montclair State University (USA)
7. Frederik Dilling, University of Siegen (Germany)
8. Jürgen Dolezal, Johannes Kepler Universität Linz (Austria)
9. Dr. Ana Donevska-Todorova, Goethe Universität Frankfurt (Germany)
10. Shereen ElBedewy, Johannes Kepler Universität Linz (Egypt)
11. Prof. Dr. Eileen Fernandez, Montclair State University (USA)
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20. Margarete Ketelsen, TU Dresden (Germany)
21. Jenny Knöppel, University of Siegen (Germany)
22. Dr. Simon Kraus, University of Siegen (Germany)
23. Dr. Diego Lieban, IFRS (Brazil)

24. Prof. Dr.-Ing. Daniel Lordick, TU Dresden (Germany)
25. Dr. Tim Lutz, Universität Landau (Germany)
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33. Dr. Felicitas Pielsticker, University of Siegen (Germany)
34. Erin Pomponio, Montclair State University (USA)
35. Katja Rasch, Deutsches Museum (Germany)
36. Bryan Sheldon, Johannes Kepler Universität Linz (Austria)
37. Dr. Gero Stoffels, University of Siegen (Germany)
38. Mathías Tejera, UdelaR (Uruguay)
39. Eva Ulbrich, Johannes Kepler Universität Linz (Austria)
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41. Dr. Miriam Voß, Technische Universität München, TUMlab im Deutschen Museum (Germany)
42. Dr. Marc Wilken, IPN Leibniz-Institut Kiel (Germany)
43. Prof. Dr. Ingo Witzke, University of Siegen (Germany)
44. Mira H. Wulff, IPN Kiel (Germany)

Information on the Book

After the symposium, a book of the contributions of the symposium will be published with Springer Publishing. All speakers (and participants) are warmly welcome to contribute with a paper.

A timeline can be seen below:

Submission of a paper	November 15, 2021
Submission of Reviews	December 15, 2021
Submission of revised papers	January 15, 2022
Publication of the book	Approx. Mai 2022

Details

Please use the template *Template International Symposium on 3D Printing in Mathematics Education, September 16-17 2021* for your contribution, which is available for download on the homepage: https://www.uni-siegen.de/fb6/didaktik/veranstaltungen/symposium3dprinting.html

Your article should not exceed 20 pages within the template. All figures and diagrams will be printed as grey scale patterns, please keep this in mind when creating your article. Please use the following information to submit your article:

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Subject: Article_"Title"_"Author(s)"

Further information will be provided at the symposium.